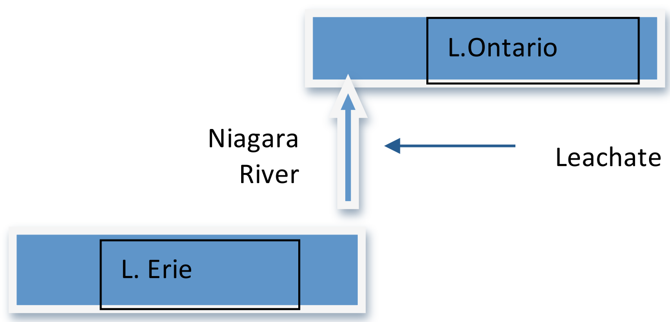
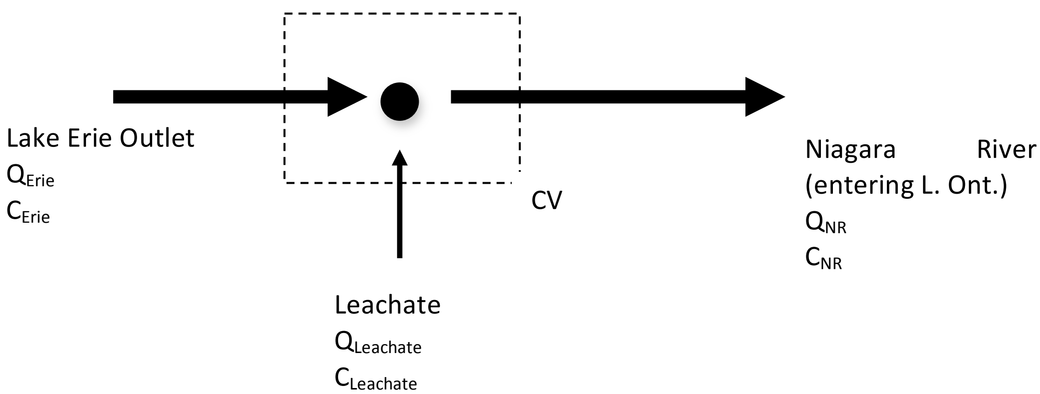
Sample Problem: Mass Balance Node

Problem:
The leachate, from the various landfills along the Niagara River, is saturated in hexachlorobenzene (the aqueous solubility of hexachlorobenzene is 0.005 g/m^3; Mackay et al., 1992) and all leachate flows into the Niagara River. The hexachlorobenzene concentration in Lake Erie is 0.05 μg/m^3 and the concentration in the Niagara River as it enters Lake Ontario is 0.4 μg/m^3. The Niagara River, as it enters Lake Ontario, has a flow of 2.1x10^{11} m^3/y. Calculate the total flowrate of leachate from the landfills.

Solution:



OR …



Assume Niagara River serves as a node. Nodes must be at steady state has they have no volume or capacity to store. Meaning nodes cannot accumulate either positively or negatively. Thus, Steady State is assumed.

	Erie Out	Leachate	Niagara River (Ontario In)
Flow (m^3/y)			2.1 x10^{11}
Concentration (μg/m^3)	0.05	5000	0.4
Mass Flow HCB (μg/y)			

Mass balance Water:

Mass Water Erie Out + Mass Water Leachate = Mass Water Ontario In

$$\rho_{w\ Erie} * Q_{Erie\ Out} + \rho_{w\ Leachate} * Q_{Leachate} = \rho_{w\ Ontario\ In} * Q_{Ontario\ In}$$

Assume the density of water is constant throughout.
(This is a very good assumption in this setting. It is important to recognize that volume flow is not conserved as a universal statement. It is mass that is conserved. Volumes are additive only when density is constant.)

Therefore,

$$Q_{Erie\ Out} + Q_{Leachate} = Q_{Ontario\ In}$$

Mass Balance Hexachlorobenzene (C_6Cl_6):

$$C_{Erie\ Out} * Q_{Erie\ Out} + C_{Leachate} * Q_{Leachate} = C_{Ont\ In} * Q_{Ont\ In}$$

Two equations & two unknowns. (Erie and Leachate flows are unknown)

Eliminate $Q_{(Erie\ Out)}$

$$C_{Erie\ Out} * (Q_{Ont\ In} - Q_{Leachate}) + C_{Leachate} * Q_{Leachate} = C_{Ont\ In} * Q_{Ont\ In}$$

$$(C_{Leachate} - C_{Erie\ Out}) * Q_{Leachate} = (C_{Ont\ In} - C_{Erie\ Out}) * Q_{Ont\ In}$$

$$Q_{Leachate} = \frac{(C_{Ont\ In} - C_{Erie\ Out}) * Q_{Ont\ In}}{(C_{Leachate} - C_{Erie\ Out})}$$

$$Q_{Leachate} = \frac{\left(0.4 \frac{\mu gHCB}{m^3} - 0.05\ \mu gHCB/m^3\right) * 2.1E11\ m^3/y}{(5000 \frac{\mu gHCB}{m^3} - 0.05 \mu gHCB/m^3)}$$

$Q_{leachate} = 1.47 \times 10^7\ m^3/y$

<u>Checks</u>

$Q_{ERIE\ OUT} = Q_{ONT\ IN} - Q_{leachate} = 2.0999 \times 10^{11}\ m^3/y = 2.1 \times 10^{11}\ m^3/y$

Mass Flow HCB Ontario In = $0.4\ \mu gHCB/m^3 * 2.1 \times 10^{11}\ m^3/y$
= 8.4e10 µgHCB/y = 84 kg HCB/y

Mass Flow HCB Erie Out = $0.05\ \mu gHCB/m^3 * 2.1 \times 10^{11}\ m^3/y$
= 1.05e10 µgHCB/y = 10.5 kg HCB/y

Mass Flow HCB Leachate = 5000 μgHCB/m^3 * 1.47 $x10^7$ m^3/y

= 7.35 $x10^{10}$ μgHCB/y

= 73.5 kg HCB/y

Summary

	Erie Out (1)	Leachate (2)	Niagara River (3) (Ontario In)
Flow (m^3/y)	2.1 $x10^{11}$	1.47 $x10^7$	2.1 $x10^{11}$
Concentration (μg/m^3)	0.05	5000	0.4
Mass Flow HCB (μg/y)	1.05 $x10^{10}$	7.35 $x10^{10}$	8.4 $x10^{10}$
(kg/y)	10.5	73.5	84

Confirms … In=Out

(Column 1 and 2 sum to Column 3 for the HCB Mass Flow rows; Note that Columns 1 and 2 do not sum to Column 3 for the Concentration row as Concentration is NOT a conserved quantity.)

(note: completing the table was not necessary to solve for the desired flow; however, it is a really effective way to summarize results and to CHECK results)

Therefore, the estimated volumetric leachate flowrate into the Niagara River is 1.5 $x10^7$ m^3/y (0.47 m^3/s).

Mackay D. *et al*., **1992**, *Illustrated Handbook of Physical-Chemical Properties and Environmental Fate of Organic Chemicals: Volume I*, Table 3.2, Lewis Publishers, Chelsea, MI.

(Note: These books, Volumes I-V, are probably among the best sources of environmentally relevant property data for organic chemicals.)