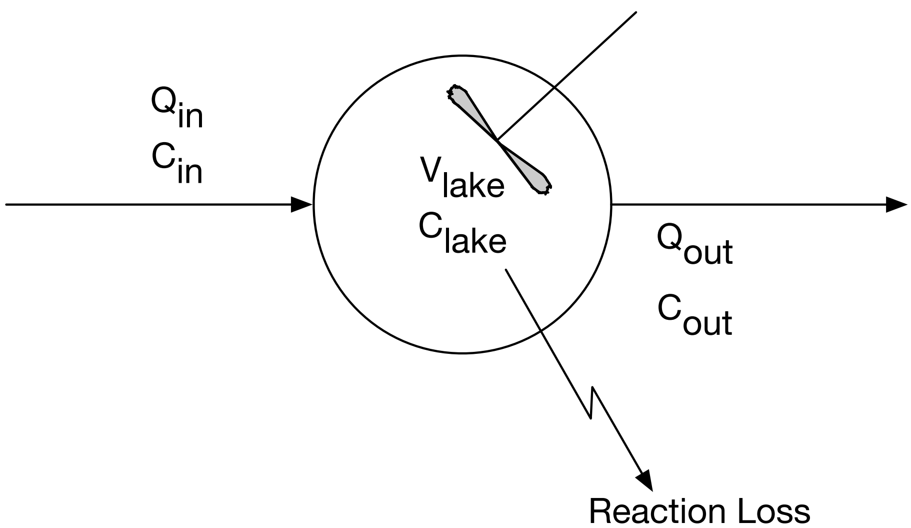
Sample Problem: CSTR with Reaction

Problem:

A lake has a contamination concentration of 16.0 μg/m^3 on January 1, 2011. The inlet river has a flow rate of 32.0 m^3/s and has a NEW (starting Jan 1, 2011) steady contamination level of 42.0 μg/m^3. The lake's volume is 3.76x10^8 m^3 and is constant. The contaminant is consumed by a reaction that is zero order with respect to the contaminant and the zero-order reaction rate coefficient is 8.2x10^{-13} g/m^3/s. What is the contaminant concentration in the lake on January 1, 2012 and January 1, 2018?

Solution:

Sketch



Lake ➔ CSTR assumption

$$V_{lake} = V = 2.76\text{x}10^8 \text{ m}^3 \text{ (constant)}$$

Constant Volume means that

$$Q = Q_{in} = Q_{out} = 32.0 \text{ m}^3/\text{s}$$

Concentration Information

$$C \text{ (on Jan 1, 2011)} = C_o = 16.0 \ \mu\text{g/m}^3 \text{ (consider Jan 1, 2011 as time zero)}$$

$$C_{in} = 42 \ \mu\text{g/m}^3 \text{ (starting Jan 1, 2011)}$$

Desire C (on Jan 1, 2012 and Jan 1, 2018) … 1 and 7 years later

Steady State Mass Balance, Lake as CV:

$$In - Out + Gen - Cons = Acc$$

Valuable to consider whether a steady state or unsteady state analysis is appropriate (i.e. whether the Accumulation term is significant). Determining the system's hydraulic retention time is useful in this context.

$$HRT = \frac{V}{Q} = \frac{3.76\text{x}10^8 \text{ m}^3}{32 \text{ m}^3/\text{s}} = 1.18\, x10^7\, s = 136\, d$$

Thus, 7 years later is the equivalent about 19 HRTs -- more than enough time to reach a new steady state. One year is approximately 2.6 HRTs – this is sufficient to approach steady state but not quite there.

Solve for the easier steady state concentration first (for the Jan 1, 2018):

No generation is evident in the context description.
Accumulation term is zero (Steady state).
Consumption is zero order reaction = kV C^0 = kV

$$Q * C_{in} - Q * C - kV = 0$$

Rearrange to solve for C

$$C = C_{in} - \frac{kV}{Q} = 42\frac{\mu g}{m^3} - \frac{8.2\, x10^{-13}\frac{g}{m^3\, s} * 3.76\, x10^8\, m^3}{32\frac{m^3}{s}} = 32.4\frac{\mu g}{m^3}$$

Thus, the concentration in the lake on January 1, 2018 is 32.4 μg/m^3.

Solve for the unsteady state concentration on January 1, 2012:

$$Q * C_{in} - Q * C - kV = V\,\frac{dC}{dt}$$

Rearrange (separation of variables)

$$\int_{Co}^{C} \frac{dC}{C - C_{in} + kV/Q} = -\frac{Q}{V}\int_0^t dt$$

Integrate

$$ln\left(\frac{C - C_{in} + kV/Q}{C_o - C_{in} + kV/Q}\right) = -\frac{Qt}{V}$$

Note: the right hand portion of the numerator and denominator in the log term is the steady state concentration previously calculated.

$$C_{in} - \frac{kV}{Q} = C_{ss} = 32.4\,\frac{\mu g}{m^3}$$

Thus

$$ln\left(\frac{C - C_{ss}}{C_o - C_{ss}}\right) = -\frac{Qt}{V}$$

Solving for C at 1 year

$$C = 31.2\,\frac{\mu g}{m^3}$$

Thus, the concentration in the lake has increased from 16.0 μg/m^3 on January 1, 2011 to 31.2 μg/m^3 on January 1, 2012 and to 32.4 μg/m3 on January 1, 2018.

(Note: the steady state concentration is less than the inlet concentration as a result of the zero order reaction)