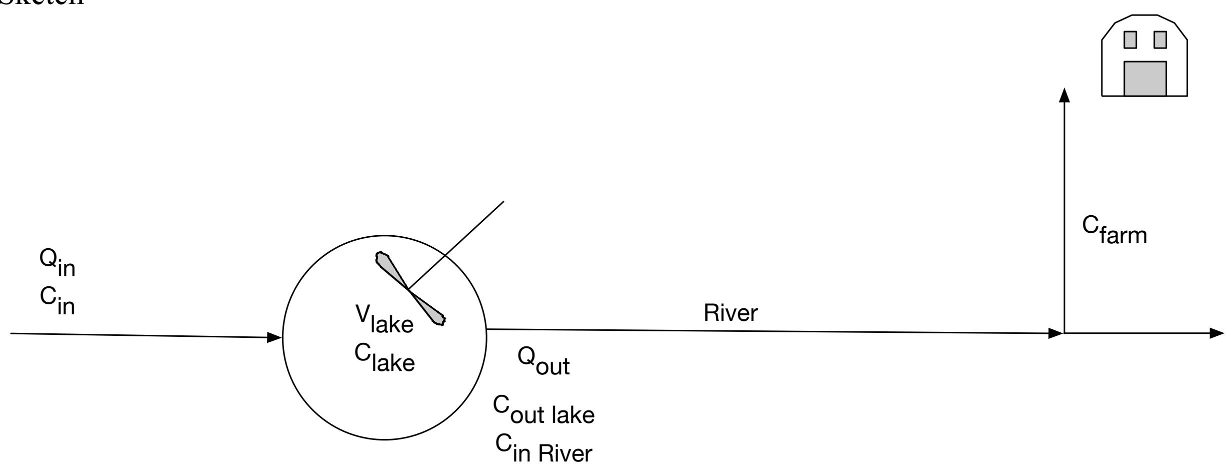
Sample Problem: Reactor System

Problem:
A single cadmium spill into a lake creates an initial concentration of 1.52 mg/m^3. The lake has a volume of 2.00x10^7 m^3 with an HRT of 42.0 days. The river flowing out of the lake has a cross sectional area of 12.0 m^2. A farm, located 15.0 km downstream of the lake, draws its irrigation water from the river. What will the concentration of the irrigation water be 3.00 days after the spill?

Solution:
Sketch



Lake ➔ CSTR assumption (stir stick symbol reinforces complete mix assumption)
River ➔ PFR assumption

In these problems, it is necessary to sort out the time lags associated with the PFR subsystem. Doing so first or early will frequently help.

River (from Lake to Irrigation Water intake):
Desire C_{farm} at t = 3.00 days (3 days after spill into lake)

C_{farm} (@ t=3.00 days) = $C_{in\ River}$ (@ t = 3.00 days MINUS HRT_{river})
(this is the fundamental character of an ideal PFR. In a non-reacting PFR, the concentration arriving downstream is the same concentration that entered the PFR one Hydraulic Retention Time, HRT, earlier)

$$HRT_{River} = \frac{V_{River}}{Q_{River}}$$

$$HRT_{River} = \frac{Length_{River} * Area_x}{Q_{River}}$$

where

Area$_x$ is the cross-sectional area of the river.

The flow rate of the river is required which is the same as the flow out from the lake. The flow out from the lake can be determined from the volume of the lake and the lake's HRT.

$$Q_{out\ lake} = \frac{V_{Lake}}{HRT_{Lake}}$$

$$Q_{out\ lake} = \frac{2.00\ x10^7\ m^3}{42.0\ days} = 4.76\ x10^5\ m^3/d$$

Thus,

$$HRT_{River} = \frac{15000\ m * 12.0\ m^2}{4.76\ x10^5\ m^3/d} = 0.378\ days$$

Therefore

$$C_{farm}(@3.00\ days) = C_{in\ River}(@3.00\ days - HRT_{River}) = C_{in\ River}(@2.62\ days)$$

And, the concentration into the river is the same as the concentration out of the lake. Thus, now ready to analyse the lake behaviour to find the concentration out at a time of 2.62 days after the spill.

Lake:

Lake CV, Mass Balance for Cadmium

$$Q * C_{in} - Q * C_{out} = V\ \frac{dC}{dt}$$

This assumes V is constant -- no information to suggest the contrary.
The spill creates an initial concentration (initial condition) in the lake but is not an ongoing source of new contaminant. Thus, C_{in} is zero. Treating the lake as a CSTR means the C_{out} and C are one and the same owing to the well-mixed assumption.

$$C_{Lake}(@t = 0) = C_o = 1.52\ mg/m^3$$

$$0 - Q * C = V\ \frac{dC}{dt}$$

$$\int_{Co}^{C} \frac{dC}{C} = \int_{0}^{2.62} \frac{-Q}{V} dt$$

$$ln \frac{C}{C_o} = -\frac{Q}{V}(2.62\ d - 0\ )$$

$$C = C_o \exp\left[\ -\frac{Q}{V}(2.62\ d)\right]$$

Therefore,

$$C = 1.43\ mg/m^3$$

Therefore the cadmium concentration leaving the lake 2.62 days after the spill occurred is 1.43 mg/m^3. This water arrives at the farm's irrigation water intake location 0.38 days later (3.00 days after the spill occurred) and thus the irrigation water concentration 3.00 days after the spill will be 1.43 mg/m^3.